A Fuzzy Set-Based Approach to Range-Free Localization in Wireless Sensor Networks

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Abstract

Localization in Wireless Sensor Networks (WSNs) refers to the ability of determining the positions of sensor nodes, with an acceptable accuracy, based on known positions of several anchor nodes. Among the plethora of possible localization schemes, the Received Signal Strength (RSS) based range-free localization techniques have attracted significant research interest for their simplicity and low cost. However, these approaches suffer from significant estimation errors due to low accuracy of RSS measurements influenced by irregular radio propagation. In order to tackle the problem of RSS uncertainty, in this work we propose a fuzzy set-based localization method as an enhancement of the ring-overlapping scheme [12]. In the proposed method, first we use a fuzzy membership function based on RSS measurements to generate fuzzy sets of rings that constrain sensor node position with respect to each anchor. Then we generate fuzzy set of regions by intersecting rings from different ring sets. Finally, we employ weighted centroid method on the fuzzy set of regions to localize the node. The results obtained from simulations demonstrate that our solution improve localization accuracy in the presence of radio irregularity, but even for the case without radio irregularity.

Keywords: Wireless Sensor Networks, Range-free localization, Received Signal Strength, Ring-overlapping, Fuzzy set theory.

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1. Introduction

Recent advances in wireless communications, low-power design, and MEMS-based sensor technology have enabled the development of relatively inexpensive and low power wireless sensor nodes. The common vision is to create a large wireless sensor network (WSN) through *ad-hoc* deployment of hundreds or thousands of such tiny devices able to sense the environment, compute simple task and communicate with each other in order to achieve some common objective, like environmental monitoring, target tracking, detecting hazardous chemicals and forest fires, monitoring seismic activity, military surveillance [1]. Most of these applications require the knowledge on the position of every node in the WSN. However, in most cases, sensor nodes are deployed throughout some region of interest without their position information known in advance. Thus, the first task that has to be solved is to localize the nodes, i.e., to find out their spatial coordinates in some fixed coordinate system. Determining the physical positions of sensor nodes is a crucial problem in WSN operation because the position information is used (i) to identify the location at which sensor readings originate, (ii) in energy aware geographic routing, and (iii) to make easier network self-configuration and self-organization. Also, in many applications the position itself is the information of interest [2].

One possible way to localize sensor nodes is to use the commonly available Global Positioning System (GPS), which offers 3-D localization based on direct line-of-sight with at least four satellites [3]. However, attaching a GPS receiver to each sensor node is highly impractical solution due to its high power consumption, high price, inaccessibility (nodes may be deployed indoors, or GPS reception might be obstructed by climatic conditions) and imprecision (the positioning error might be of 10-20m) [4]. A number of localization systems and algorithms have been proposed recently specifically for WSNs, which are generally classified into range-based and range-free localization schemes. The range-based localization depends on the assumption that sensor nodes have the ability to estimate the distance or angle to other nodes by means of one or more of the following measurements: received signal strength (RSS), time of arrival (TOA), time difference of arrival (TDOA), and angle of arrival (AOA) [5][6][7][8][9]. Although the range-based schemes typically provide a lower estimation error than the range-free schemes, they require installation of specific and expensive hardware (e.g., directive antennas) to obtain relatively accurate distance (or angle) measurements and have weakness in the noisy environments.

In contrast to range-based technique, the range-free scheme enables sensor nodes to estimate their locations without relying on distance/angle measurements [2][4][10][11]. Such techniques generally require numerous anchors (location-aware nodes) which enable location-unknown sensor nodes to determine their locations by exploiting the radio connectivity information among nodes, or by comparing their RSS measurements with those supplied by anchors or nearby nodes. Range-free solutions use only standard features found in most radio modules as hardware means for localization, thus providing more economic and simpler location estimates than the range-based ones. On the other hand, the results of range-free methods are not as precise as those of the range-based methods.

In this paper we propose a distributed range-free localization technique, called Fuzzy-Ring, which utilizes the received signal strength information to estimate the relative position of a sensor node with respect to a small number of randomly distributed anchors. Similar to other area-based localization methods [11][12][13], Fuzzy-Ring uses beacons broadcasted by anchors to isolate a region of the localization space where the sensor node most probably resides. Like in ROCRSSI algorithm [12], the localization region is defined as the intersection area of overlapping annular rings which constrain the position of the sensor node with respect to each anchor. The rings are generated by comparison of the signal strength a sensor node receives from a specific anchor and the signal strength other anchors receive from the same anchor. The novelty of our localization scheme is to represent overlapping rings as fuzzy sets with ambiguous boundaries. The use of the fuzzy set theory is motivated by the fact that RSS measurements are usually inaccurate due to a number of factors such as multipath propagation, reflection, interference and shadowing among others. Such irregularity of the radio propagation creates non-circular ring borders and might induce a significant localization error when a binary decision-making model ("in-ring" versus "out-of-ring") is employed, like in ROCRSSI. In our approach, we use fuzzy membership functions based on the RSS information to represent the degree to which a sensor node location is within different rings, which helps to improve localization estimations, especially for sensor nodes located in proximity to ring boundaries.

The rest of the paper is organized as follows: Section 2 introduces localization based on comparison of RSS and discuses how fuzzy set theory can be applied in this range-free approach. Section 3 presents the proposed localization approach. Results for the simulations are shown in Section 4. Finally, Section 5 concludes this paper.

2. Localization based on comparison of RSS

RSS–based range–free algorithms only rely on the assumption that the RSS is a decreasing function of the distance between transmitter and receiver. For example, if the straight of the beacon signal that sensor node S receives from anchor A is smaller than the straight of the same signal received by anchor B, then S can conclude that it is closer to A then to B. A number of distance constraints, produced after a series of such comparisons, will enable the sensor node to confine its position within a limited area of the localization space.

Let consider a network with n=3 anchors placed at fixed known positions in 2-D localization space shown in Fig. 1. Around each anchor the set of n-1 concentric circles is placed. Radius of every circle equals distance between center anchor and one of n-1 remaining anchors. Each set of concentric circles partitions the localization space into n rings numbered from 1 to n. The ring 1 corresponds to the area of the innermost circle, while the ring n-1 corresponds to the outside area of the last circle. A localization region is the intersection area of rings from different ring sets, while its area-code is the sequence of the intersecting ring numbers, i.e. ring ranks. For example, shaded area in Fig. 1 represents the region with area-code (2, 1, 2), i.e. the region which is obtained by intersecting rings 2, 1 and 2 of anchors A_1, A_2, and A_3, respectively. Note that for each point p in this region, the area-code (2, 1, 2) defines the following set of distance-based constraints:

$$d(A_1, A_2) \le d(A_1, p) \le d(A_1, A_3)$$
$$d(A_2, A_2) \le d(A_2, p) \le d(A_2, A_3)$$
$$d(A_3, A_2) \le d(A_3, p) \le d(A_3, A_1)$$

where $d(x, y)$ denotes the distance between two anchors/nodes.

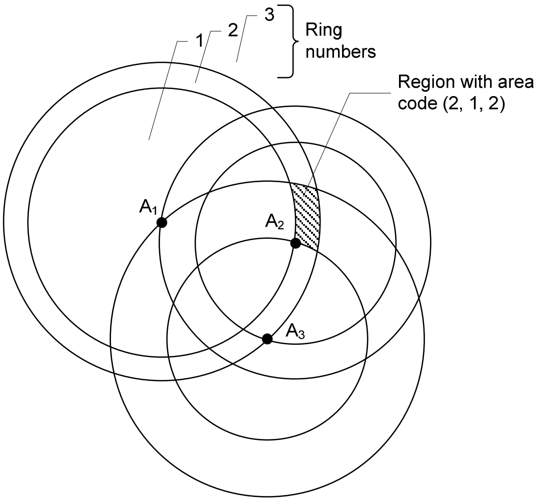


Fig. 1. Rings, regions and area codes.

In an idealistic physical environment, RSS measured at a point further from an anchor is always smaller then RSS measured at a point that is closer to the same anchor. As a consequence, if we use RSS information for the regionalization of the localization space, the resulting set of regions, i.e. the set of region area-codes, will be the same as one obtained by using distance information. This means that a sensor node will always be able to locate itself into the correct localization region by using the comparison of RSS values, only. For example, the following set of RSS-based constraints will confine the location of the sensor node S inside the region (2, 1, 2) of the network given in Fig. 1:

$$rss(A_1, A_2) \geq rss(A_1, S) \geq rss(A_1, A_3)$$
$$rss(A_2, A_2) \geq rss(A_2, S) \geq rss(A_1, A_3)$$
$$rss(A_3, A_2) \geq rss(A_3, S) \geq rss(A_1, A_1)$$

where $rss(x,y)$ denotes the strength of the signal broadcasted by anchor x as measured by anchor/node y.

However, the radio propagation is usually not homogenous in all directions because of the presence of multi-path fading and different path losses depending on the direction of propagation. As a result, the ordering of the anchors based on comparison of RSS values might not be identical with their ordering based on comparison of Euclidean distances, which might induce localization errors. Let consider a partial distance-based regional map of the network configuration with four anchors given in Fig. 2. A sensor node S, depicted with square mark, is located in the region with area-code (3, 1, 1, 3). However, due to inaccurate RSS measurements, S may easily came up with the wrong decision regarding its regional area-code. For example, in order to correctly locate itself into ring 3 of the anchor A_4, S should measure lower strength of the beacon signal broadcasted by A_4 then anchor A_3. However, because of the irregularity in radio propagation, there is a chance for S to measure a higher RSS value then A_3. Such an imprecision will cause the sensor node S to choose the area-code (3, 1, 1, 2) instead of the correct one (3, 1, 1, 3). As S is closer to the boundary of the A_4's ring 3 the chance for the wrong ring selection is larger. Note that area code (3, 1, 1, 2) is *valid*, but *wrong*. Under some circumstances, S may pick an area-code that does not even exist in distance-based regional map, i.e. it may select so called *invalid* area-code. For example, an incorrect ring selection with respect to anchor A_2 could result in the area-code (3, 2, 1, 3). It is easy to see that this area-code does not exist in the given regional map since 3rd rings of anchors A_1 and A_4 intersect in two regions, only, i.e. in regions (3, 1, 1, 3) and (3, 4, 4, 3).

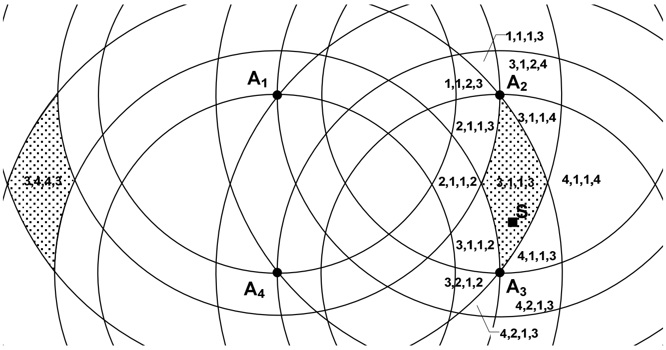
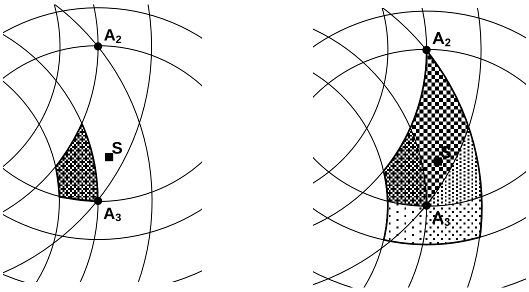


Fig. 2. A segment of distance-based regional map.

In ROCRSSI algorithm, the problem of incorrect ring/region selection is solved by choosing the valid region (or regions) with the maximum number of intersecting rings. However, in this approach, a small radio irregularity in the proximity of ring boundaries may lead to the large localization error, as illustrated in Fig. 3(a). In order to overcame the uncertainty of the RSS and the nonlinearity between the RSS and the distance, in this work we suggest a different approach based of the fuzzy set theory. We use fuzzy sets to model relationship between localization regions and the RSS information available to the sensor node. When comparing its RSS measurements with those of anchors, the sensor node will not select one region only, but it might choose two or more regions each with different *weight* representing certainty of the decision. This concept is illustrated in Fig. 3(b). Different shades of gray indicate different weighs of the selected regions. Finally, the center of gravity of the shaded area is used as the final location estimation.



(a) (b)

Fig. 3. ROCRSSI vs. fuzzy selection of localization regions.

3. Fuzzy-Ring Localization Algorithm

In this section, we describe our novel fuzzy set-based range-free localization scheme, which we call Fuzzy-Ring. Fuzzy-Ring requires a heterogeneous wireless sensor network composed of two sets of static nodes distributed across a planar sensing field: the set of anchors, i.e. the nodes whose locations are known, and the set of sensor nodes, i.e. the nodes whose locations are to be determined. For simplicity and ease of presentation we limit the sensing field to 2-D, but with minor modifications our algorithm is capable of operating in 3-D. Anchor nodes, or nodes with *a prior* knowledge of their locations in the sensing field (obtained via GPS or other means such as pre-configuration), serve as reference points, broadcasting beacon messages. They are assumed to be sparse and randomly located. Both anchors and sensor nodes are equipped with omni-directional antennas and RF transceivers with built-in RSS indicator (RSSI) circuitry. Anchors are assumed to have a larger communication range than normal sensor nodes so that their beacons can reach all wireless nodes in the network. The Fuzzy-Ring algorithm proceeds in four phases: 1) Beacon exchange, 2) Distance-based regionalization, 3) RSS-based regionalization, and 4) CoG calculation. These steps are performed at individual nodes in a purely distributed fashion.

3.1. Beacon exchange

The Fuzzy-Ring algorithm requires every anchor to periodically send out beacon message including its own ID. While receiving a beacon every anchor and sensor node samples the received signal straight (RSS) and stores measured value together with the ID of the transmitting anchor. At the end of this period, all anchors broadcast so-called localization messages to all sensor nodes. Localization message includes anchor's ID and its location information, along with a vector containing the recorded RSS values of beacons it received from other anchors. We assume that anchors are synchronized so that their beacon and localization message transmissions do not overlap in time. Note that after beacon exchange phase, each sensor node *S* knows not only locations of all anchors, but also RSS values measured between each pair of anchors as well as between each anchor and S, i.e.:

- Vector $[l_i]$, $i \in \{1,...,n\}$ of anchors locations wherein $l_i = (ax_i, ay_i)$ is 2-D coordinate of anchor A_i in the sensor field.
- Vector $[x_i]$, $i \in \{1,...,n\}$ of sensor node's RSSI readings wherein x_i is the strength of the beacon signal that S received from anchor A_i.
- Matrix $[r_{i,j}]$ of anchors' RSSI readings wherein $r_{i,j}$ is the strength of the signal that anchor A_j received form anchor A_i.

3.2. Distance-based regionalization

After collecting enough information, a sensor node can start the localization process. The first step in this process is to create a distance-based regional map of the localization space by using the known locations of all anchors. Let $AS = \{A_i\}$, $i = 1... n$, be the set of anchors. Taking A_i as the center anchor, elements in AS can be arranged into the *anchor sequence* $Q_i = (a_0 = A_i, a_1, ..., a_{n-1}, a_n = F)$ ordered by distance to A_i, wherein $a_j \in AS \setminus \{A_i\}$, $j \in \{1,...,n-1\}$, and F is a fictive anchor placed in infinity. Thus, $d(A_i, a_j) \leq d(A_i, a_{j+1})$, $j \in \{1,...,n\}$. Each anchor sequence Q_i, $i \in \{1,...,n\}$ defines an ordered sequences of distance-intervals $T_i = ([0, d(A_i, a_1)], ..., [d(A_i, a_j), d(A_i, a_{j+1})], ..., [d(A_i, a_{n-1}), \infty])$. For any point p in localization space and any anchor A_i, there is exactly one distance-interval in T_i such that $d(A_i, a_j) \leq d(A_i, p) \leq d(A_i, a_{j+1})$. The *rank* of point p with respect to anchor A_i, written as $rank_i(p)$, is the ordinal number of the corresponding distance-interval in T_i. *Area-code* of point p, written as $C(p)$, is defined as the sequence of its ranks with respect to all anchors, i.e. $c(p) = (rank_1(p), ..., rank_n(p))$. Note that the order in which the ranks are written in an area-code is determined by a predefined order of anchor IDs. *Localization region*, R, with area-code $C(R) = C$ is the set of all points in the localization space with the same area-code, i.e. $R = \{p \mid c(p) = C\}$. *Distance-based regional map*, denoted as D, is the set of area-codes of all regions identified in the localization space.

For the purpose of the proposed localization algorithm, each region is represented by the following two attributes: (a) the area-code, and (b) the center of gravity (CoG). CoG of region R is defined as the average location of all points in R. Since the analytical procedure for finding regions and their CoGs is rather involved for resource constrained sensor nodes due to the large number of floating point operations, we adopted an approximate approach based on *grid-scan* algorithm [11][12]. In this algorithm, the localization space is divided into uniform grid of M x M points, and the area-code is determined at each point in the grid. Initially, the distance-based regional map, D, is empty. In order to identify localization regions, the grid is scanned, point-by-point. For each grid point p with coordinates (px, py) the area-code $c(p)$ is first determined based on its Euclidean distances to the anchors and pre-determined anchor sequences, and then the regional map D is searched for that code. If the area-code $c(p)$ is not found, it is inserted into the map D, and the new region R is created with the code $c(p)$. Four variables, C, X, Y, and P, are associated with each region, representing its area-code, the sum of x-coordinates, the sum of y-coordinates, and the total number of grid points in the region, respectively. When the new region is created, its variables are initialized to $C = C(p)$, $P = 1$, $X = p_x$, and $Y = p_y$. Otherwise, if D already contains the area-code $C(p)$, variables X, Y, and P of that region are updated; i.e. $X = X + p_x$, $Y = Y + p_y$ and $P = P + 1$. After grid scanning is completed, the region CoG coordinates are calculated as $(\frac{X}{P}, \frac{Y}{P})$.

3.3. RSS-based regionalization

In this algorithm step, sensor node creates a *RSS-based regional map* by comparing its own RSSI readings with those gathered from anchors. Assume that RSS measurements are in the range I = [0, RSS_max], and denote with $r_{i,j}$ the RSS value of the beacon signal sent by anchor A_i as measured by j^{th} anchor in anchor sequence $Q_i = (a_0 = A_i, a_1, ..., a_{n-1}, a_n = F)$. We also assume that $r_{i,0}$ = RSS_max, and $r_{i,n} = 0$. The value of $r_{i,j}$ splits the RSS range into two disjoint sub ranges: $LT_{i,j} = [0, r_{i,j}]$, and $GT_{i,j} = [r_{i,j}, RSS\_max]$. Note that $LT_{i,0} = GT_{i,n} = I$. The *RSS-interval* with rank j in respect to anchor A_i, denoted as $RI_{i,j}$, represents the sub range of RSS values between those measured by two successive anchors, $a_{i,j}$ and $a_{i,j+1}$, in anchor sequence Q_i. In the case of monotonic attenuation of radio signal, i.e. when $r_{i,j} \geq r_{i,j+1}$, RSS-interval is defined as $RI_{i,j} = [r_{i,j}, r_{i,j+1}]$. However, due to the irregular radio propagation, the monotonic characteristic might not be always satisfied, and it might happen that $r_{i,j} \leq r_{i,j+1}$, i.e. $RI_{i,j} = [r_{i,j+1}, r_{i,j}]$. Thus, the RSS-interval can be defined as:

$$RI_{i,j} = \begin{cases} LT_{i,j} \cap GT_{i,j} & if \quad r_{i,j} \geq r_{i,j+1} \\ GT_{i,j} \cap LT_{i,j} & if \quad r_{i,j} < r_{i,j+1} \end{cases} \tag{1}$$

Fig. 4 shows the relationship between distance-intervals and RSS-intervals along one particular anchor sequence. RSS values of A_1's beacon signal as measured by three different sensors, S_1, S_2, and S_3 are denoted with rs_1, rs_2, and rs_3, respectively. Note that in 2-D localization space each distance-interval corresponds to one ring centered in anchor A_1. Note also that due to the anomaly in the radio propagation, anchor A_2 measures a larger RSS value than A_3 introducing overlapping of RSS-intervals. Using the "rigid" comparison of RSS values both sensors S_1 and S_2 will be localized within the ring $A_1 - A_4$ since $rs_1, rs_2 \in RI_{1,1}$. Although we may have a high degree of confidence in this decision as sensor S_1 is concerned, the real location of sensor S_2 is much more uncertain due to its proximity to the boundary between RSS intervals $RI_{1,1}$ and $RI_{1,2}$. The radio irregularity makes the position of sensor node S_3 even more uncertain since its RSS measurement, rs_3, fits in three RSS intervals, $RI_{1,2}$, $RI_{1,3}$ and $RI_{1,4}$.

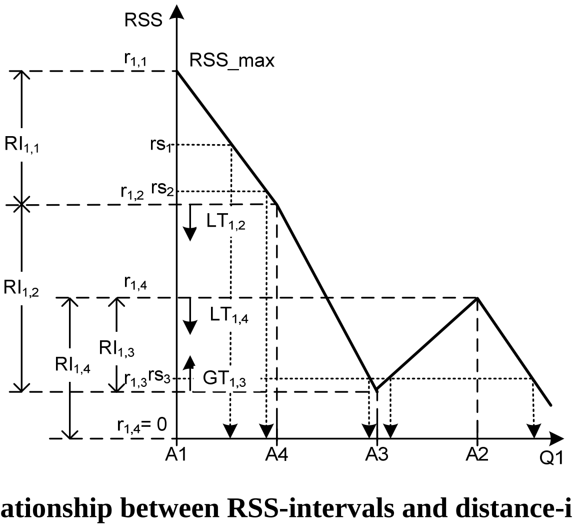


Fig. 4. Relationship between RSS-intervals and distance-intervals.

In order to minimize the effect of the uncertainty in RSS values on transition from RSS-based to distance-based domain, we explicitly incorporate the uncertainty in the modeling of RSS intervals using the concepts of fuzzy set theory. First, we represent RSS sub ranges, $LT_{i,j}$ and $GT_{i,j}$, as fuzzy sets $\widetilde{LT}_{i,j} = \{(x, \mu_{LT}^{i,j}(x)) | x \in I\}$ and $\widetilde{GT}_{i,j} = \{(x, \mu_{GT}^{i,j}(x)) | x \in I\}$, where $\mu_{LT}^{i,j}(x)$, and $\mu_{GT}^{i,j}(x)$ represent membership functions defined as:

$$\mu_{LT}^{i,j}(x) = \begin{cases} 1 & if \quad x < r_{i,j}(1-P) \\ \dfrac{1}{2P}\left(\dfrac{x}{r_{i,j}} - 1\right) - \dfrac{1}{2} & if \quad r_{i,j}(1+P) \leq x \leq r_{i,j}(1-P) \\ 0 & if \quad x > r_{i,j}(1+P) \end{cases}$$

$$\mu_{GT}^{i,j}(x) = 1 - \mu_{LT}^{i,j}(x)$$

A graphical interpretation of membership functions $\mu_{LT}^{i,j}(x)$, and $\mu_{GT}^{i,j}(x)$ is given in Fig. 5. Parameter *P*, which we call the level of fuzzification, is involved to enable adaptation to various levels of radio propagation irregularity. The value of $P \in [0,1]$ controls the width of fuzzy region of the membership functions. Functions $\mu_{LT}^{i,j}(x)$, and $\mu_{GT}^{i,j}(x)$ are used by a sensor node when it compares the straight of the beacon signal it received from anchor A_i, x, with the RSS value $r_{i,j}$ of the same beacon measured by j^{th} anchor in the anchor sequence headed in A_i. When x is outside the fuzzy region, the outcome of this comparison is strictly "greater then" or "less then" describing the full membership in one of two RSS sub-ranges, $LT_{i,j}$ or $GT_{i,j}$. On the other hand, when x is in the fuzzy region, the result of the comparison is a partial membership in both RSS sub-ranges.

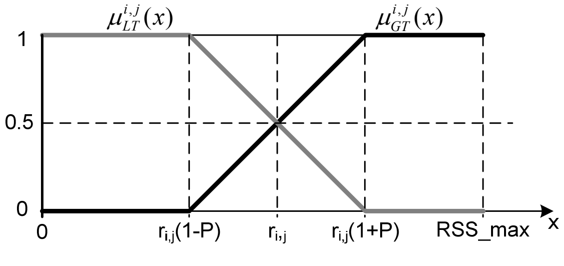


Fig. 5. Membership functions.

Fuzzy set $\widetilde{RI}_{i,j} = \{(x, \mu_{RI}^{i,j}(x)) | x \in I\}$ representing RSS-interval $RI_{i,j}$ is derived according to (1) by applying the fuzzy intersection operation on fuzzy sets $\widetilde{LT}_{i,j}$ and $\widetilde{GT}_{i,j}$. For membership function $\mu_{RI}^{i,j}(x)$ we use:

$$\mu_{RI}^{i,j}(x) = \begin{cases} \mu_{LT}^{i,j}(x) + \mu_{GT}^{i,j+1}(x) - 1 & if \quad r_{i,j} \geq r_{i,j+1} \\ \mu_{GT}^{i,j}(x) + \mu_{LT}^{i,j+1}(x) - 1 & if \quad r_{i,j} < r_{i,j+1} \end{cases} \tag{2}$$

Note that expression (2) is known in fuzzy set theory as bounded product (or bold intersection) [14]. A sensor node uses function $\mu_{RI}^{i,j}(x)$ to determine the degree of its membership in j^{th} ring placed around anchor A_i.

Given vector $X = (x_1, x_2, ..., x_n)$ of sensor node's RSS measurements, the fuzzy-ring set with respect to anchor A_i, written as $\widetilde{RI}_i$, is the set of ranks of all RSS-intervals with non-zero degree of membership in respect to anchor A_i, i.e.,

$$\widetilde{RI}_i = \{(j,\ \mu_{RI}^{i,j}(x)) |\ \mu_{RI}^{i,j}(x) > 0\}, i = 1, \dots, n$$

Note that elements of $\widetilde{RI}_i$ are integers in range [1, n]. Note also that $\widetilde{RI}_i \subseteq \mathscr{P}(\{1, 2, ..., n\})$, where $\mathscr{P}$ denotes a power set. Cardinality of a fuzzy-ring set depends on several factors, such as the number of anchors, the value of parameter P, and the level of radio propagation irregularity.

The *RSS-based regional map* is Cartesian product of fuzzy-ring sets:

$$\widetilde{RM} = \widetilde{RI}_1 \times \widetilde{RI}_2 \times \dots \times \widetilde{RI}_n = \{(C, \mu_{RM}(C))\},$$

where $C = (j_1, j_{2,} \dots, j_n)$ is area-code, and $\mu_{RM}(C) = \mu_{RI}^{1,j_1}(x_1) \cdot \mu_{RI}^{2,j_2}(x_2) \cdot \dots \cdot \mu_{RI}^{n,j_n}(x_n)$.

Elements of $\widetilde{RM}$ are region area-codes. The degree of membership of an area-code in RSS-based regional map is derived by multiplying degrees of sensor node membership in all rings that intersect in the corresponding region.

The last operation of the fuzzification process filters out all invalid area-codes from the RSS-based regional map. The output is the fuzzy set of area-codes, $\widetilde{FR}$, that contains all area-codes of $\widetilde{RM}$ that also belong to the distance-based regional map, *D*, i.e.

$$\widetilde{FR} = D \cap \widetilde{RM} = \{(C, \mu_{RM}(C)) | C \in D, C \in \widetilde{RM}\}$$

Note that degrees of membership of area-codes in $\widetilde{FR}$ are the same as in $\widetilde{RM}$. In some rare cases it may happen that $\widetilde{FR}$ does not contain any area-code. In such situations, there are two options: to left the sensor node unknown (i.e. not-localized), or to repeat the fuzzification process with a larger value of parameter *P*. We implement the second one.

3.4. CoG Calculation

Given the fuzzy set of area-codes, $\widetilde{FR}$, the goal of the final step of the localization process is to find the crisp real value that represent the estimated location of the sensor node. Note that two numeric attributes are associated with each area-code in $\widetilde{FR}$: (a) the CoG of the corresponding localization region, and (b) the degree of membership in $\widetilde{FR}$. We use Center of Area method (CoA) to produce final location estimation:

$$Estimated\_location = \left\{ \frac{\sum_{C \in \widetilde{FR}} x_{COG}(C) \cdot \mu_{RM}(C)}{\sum_{C \in \widetilde{FR}} \mu_{RM}(C)}, \frac{\sum_{C \in \widetilde{FR}} y_{COG}(C) \cdot \mu_{RM}(C)}{\sum_{C \in \widetilde{FR}} \mu_{RM}(C)} \right\}$$

where $x_{COG}(C)$ and $y_{COG}(C)$ are *x*- and *y*-coordinate of CoG of the region with area-code *C*.

4. Simulation Results

We implement Fuzzy-Ring localization algorithm in a custom WSN simulator build in C++, and conducted several experiments to evaluate its performances. In addition, we compare our results to the ROCRSSI algorithm. Our evaluation is based on the simulation of a benchmark set of 60 different network configurations categorized into six subsets of ten networks with the same number of anchors. We analyze network configurations with 3 to 8 anchors. All networks in one subset are created by varying positions of n anchors within the same basic setup of 200 sensor nodes randomly deployed in a circular area of 100 m in diameter. In our simulations, we intend to illustrate the impact of the number of anchors, the degree radio propagation irregularity, and the level of fuzzification on the localization error. The performance of two localization methods is evaluated using the location estimation error defined as $(d/D)*100\%$, where d is the Euclidian distance between the real location of a sensor node and its estimated location, and D is the diameter of the localization area.

Radio propagation model

We adopt the Degree Of Irregularity (DOI) radio propagation model introduced in [11] and subsequently extended in [12]. In this model, the signal strength is defined as $C \times K(\theta)/d^2$, where C is a constant, d is the distance between the receiver and the transmitter, and $K(\theta)$ is the coefficient representing the difference in path loss in different directions. $K(\theta)$ is calculated according to (3) where $\theta \in [0,360^o]$, *rand* is a random number uniformly distributed in range [-1,1], $s = \lfloor \theta \rfloor$, and $t = \lceil \theta \rceil$ [12]. The parameter *DOI* is used to denote the irregularity of the radio pattern. It is defined as the maximum signal straight variation per unit degree change in the direction of radio propagation. When *DOI* value is 0, there is no variation in the signal straight which results in a perfectly circular radio model. When *DOI* > 0, large *DOI* values represent large variation of radio irregularity. Examples of two characteristic DOI values of this irregular radio pattern model are shown in Fig. 6.

$$K(\theta) = \begin{cases} 1 & \theta = 0 \\ K(\theta - 1) + rand \times DOI & \theta \text{ is positive integer} \\ K(t) + (\theta - s) \times (K(t) - K(s)) & otherwise \end{cases} \quad (3)$$

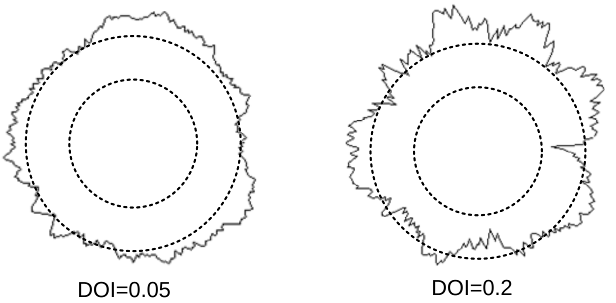


Fig. 6. Irregular radio patterns for different values of DOI.

Localization Error when varying P

In order to analyze the impact of the fuzzification level, expressed by the value of parameter *P*, on the localization error we simulate Fuzzy-Ring algorithm under various degrees of radio propagation irregularity in a network with *NA*=5 anchors. The value of parameter *P* defines the width of the boundary region of fuzzy membership functions. When *P* value is 0, the fuzzification is practically switched-off by forcing selection of one ring per anchor, only. When *P* is greater than 0, the chance of selecting multiple rings per anchor increases. In this way, the value of *P* directly influences the cardinality of the fuzzy set of regions. Fig. 7 shows the location error as a function of *P* for four characteristic values of *DOI*.

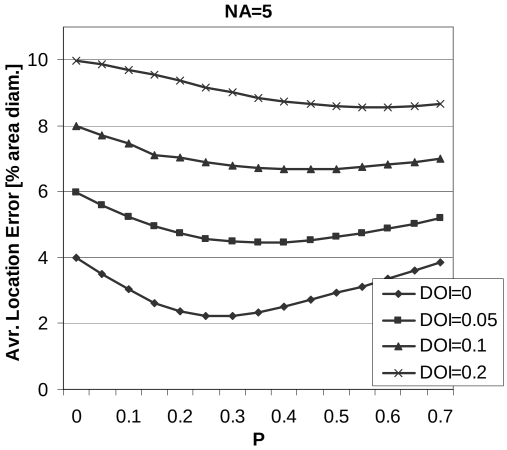


Fig. 7. Localization error vs. level of fuzzification (P) for four different degrees of radio propagation irregularity (DOI).

As can be seen from Fig. 7, the fuzzy approach is beneficial even in the regular (i.e. circular) radio propagation scenario (i.e. when *DOI* value is 0). The regular radio propagation pattern guaranties an ideal "1-1" matching between regions in distance- and RSS-based regional maps enabling the Fuzzy-Ring algorithm to always select the correct localization region, even without fuzzification (*P*=0). The only source of localization errors is due to the distance between the CoG of the region and the real location of the sensor node. When $P > 0$, the result of fuzzification process is the fuzzy set of regions that includes multiple localization regions with different weights. This can move the CoG toward the real location of the sensor node. On the other hand, an irregular radio propagation pattern ($DOI > 0$) creates non-circular region borders causing a non-ideal mapping between distance- and RSS-based localization regions. Without the fuzzification (*P*=0), localization error is influenced not only by the inter-region errors but also by the wrong ring selection. On the other hand, with the fuzzification switched-on (*P*>0) the fuzzy set of region will likely include the correct region along with several nearby regions which will partially compensate the inaccuracy of RSS measurements.

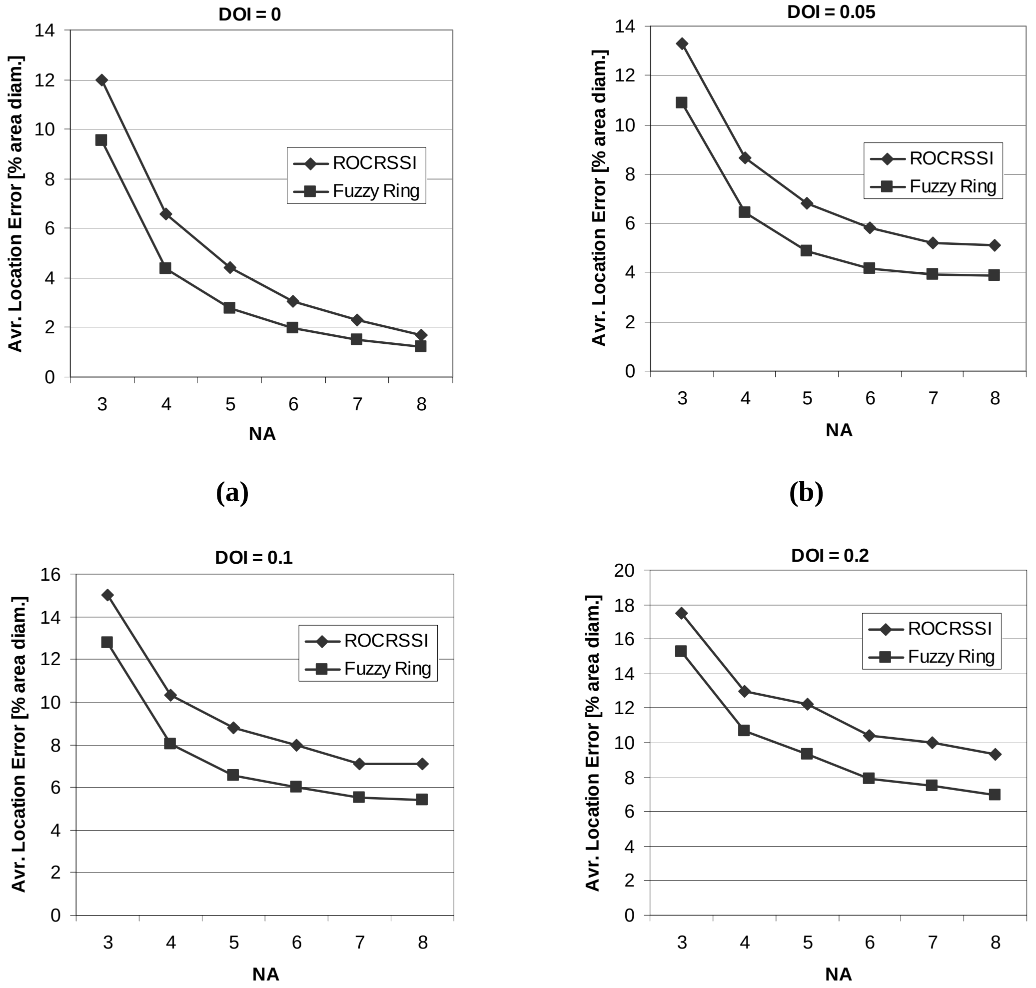
As can be seen from Fig. 7, for every value of *DOI* there is an optimal value of the parameter *P*, P_{opt}, for which the Fuzzy-Ring algorithm achieves the best performances. For analyzed range of *DOI* values, the P_{opt} ranges from 0.25, for *DOI*=0, to 0.65, for *DOI*

= 0.2. Based on the above results, we suggest that the value of *P* should be set to 0.3 – 0.4 in order to achieve the minimal additional localization error caused by non-optimal selection of *P*. For example, by using *P*=0.35, independently of *DOI*, the maximal additional localization error is less than 10% in respect to P_{opt} over the analyzed range of *DOI*.

Localization Error when varying number of anchors

In this experiment, we study the influence of the number of anchors, *NA*, on localization error. We apply both ROCRSSI and Fuzzy-Ring algorithms to all network configurations in the benchmark set with $DOI \in \{0, 0.05, 0.1, 0.2\}$ and P set to 0.35. Fig. 8 shows the location errors as a function of the number of anchors deployed. Each data point in these graphs represents the average value of 20.000 localization trials. First, for every *NA* we simulate 10 different anchor configurations within the network of 200 sensor nodes. Second, for each anchor configuration, 10 runs with different random seeds for *DOI* were executed.

From Fig. 8, we can observe that Fuzzy-Ring always outperforms ROCRSSI in terms of localization error, no matter whether the radio propagation is regular or irregular. For example, Fig. 8(b) shows that when we set the number of anchors at 6, Fuzzy-Ring achieves 0.04D accuracy, which is 28% more accurate than the ROCRSSI. Thus, Fuzzy-Ring algorithm enables us to deploy a smaller number of anchors to obtain the same level of performances as with ROCRSSI. For example, when *DOI* is 0.05, the Fuzzy-Ring only needs 5 anchors to achieve the same localization error as ROCRSSI with 8 anchors. However, it is important to note that Fuzzy-Ring is more computationally intensive then ROCRSSI. Based on the above results, we suggest that the optimal number of anchors should be 5 or 6 because deploying a larger number of anchors results in marginal improvement of localization accuracy, only.



(a) (b)

(c) (d)

Fig. 8. Localization error vs. number of anchors for: (a) DOI = 0; (b) DOI = 0.05; (c) DOI = 0.1; (d) DOI = 0.2.

Localization Error when varying DOI

In this experiment, the results of which are shown Fig. 9, we quantify the degree of localization performance degradation in the presence of radio irregularity. We conduct the analysis on the network configuration with 5 anchors under various degrees of radio propagation irregularity (*DOI*). For Fuzzy-Ring algorithm we set the level of fuzzification to *P*=0.35. From Fig. 9, we observe that with the increase of *DOI* values, the localization error keeps increasing. When *DOI* is 0, the radio propagation patter is circular and the localization error is 0.04*D*, for ROCRSSI, and 0.023*D*, for Fuzzy-Ring. But when the *DOI* increases to 0.2, the radio propagation patter becomes very irregular and the localization error increases to 0.12*D*, for ROCRSSI, and 0.089*D*, for Fuzzy-Ring. Note that Fuzzy-Ring outperforms ROCRSSI in the whole range of analyzed *DOI* values reducing the location error for 0.017*D*, when *DOI* is 0, up to 0.03*D*, when *DOI* is 0.2.

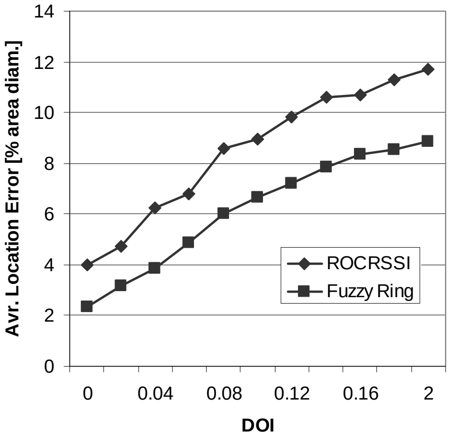


Fig. 9. Localization error vs. degree of radio propagation irregularity for ROCRSSI and Fuzzy-Ring algorithms.

5. Concussion

Many applications of wireless sensor networks depend on accurate determination of the positions of all network nodes. In this paper, we describe and investigate RSS-based range-free localization method, called Fuzzy-Ring. The novelty of our scheme is to combine ROCRSSI, a ring-overlapping approach originally proposed in [12], and the fuzzy set theory for performing sensor localization. Fuzzy set theoretical approach helps to manage uncertainty associated with RSS more efficiently with less number of anchors. Simulation results show that Fuzzy-Ring performs better than ROCRSSI in terms of localization accuracy by about 15-30% under different number of anchors and degrees of radio propagation irregularity. We also show that Fuzzy-Ring improves range-free localization under ideal radio propagation model.

This paper does not consider the way to adapt the level of fuzzification to the varying degree of radio propagation irregularity. This remains for a future work. In our future work, we would also like to test the proposed scheme under more network scenarios such as limited radio range of anchors, and to study the effect of topology of anchor nodes on localization error.

6. References

[1] F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, ”Wireless sensor networks: A survey”, *Computer Networks*, vol. 38, no. 3, pp. 393–422, 2002.

[2] N. Bulusu, J. Heidemann, and D. Estrin, “GPS-less low-cost outdoor localization for very small devices”, *IEEE Personal Communications*, vol. 7, no. 5, pp. 28 -34, Oct. 2000.

[3] B. H. Wellenhoff, H. Lichtenegger, and J. Collins, *Global positions system: theory and practice*, *Fourth Edition*, Springer Verlag, 1997.

[4] D. Nicolescu, B. Nath, “DV based positioning in ad hoc networks”, *Journal of Telecommunication Systems*, vol. 22, no. 1, pp. 267–280, 2003.

[5] G. Mao, B. Fidan, and B. Anderson, “Wireless Sensor Networks Localization Techniques”, *Computer Networks*, vol. 51, no. 10, pp. 2529-2553, 2007.

[6] D. Niculescu, B. Nath, “Ad hoc positioning system (APS) using AOA”*, In Proc. of the 22nd Annual Joint Conference of the IEEE Computer and Communications Societies*, San Francisco, USA, pp. 1734−1743, 2003.

[7] Y. Kwon, K. Mechitov, S. Sundresh, W. Kim, and G. Agha, “Resilient Localization for Sensor Networks in Outdoor Environments”, *In Proc. of 25th IEEE International Conference on Distributed Computing Systems (ICDCS)*, pp. 643-652, 2005.

[8] A. Savvides, C. C. Han, and M. B. Srivastava, “Dynamic fine-grained localization in ad-hoc networks of sensors”, *In Proc. of the seventh annual international conference on Mobile computing and networking (MobiCom 2001)*, pp. 166–179, 2001.

[9] P. Bahl, and V. N. Padmanabhan, “RADAR: An in-building RF-based user location and tracking system”, *INFOCOM 2000. Nineteenth Annual Joint Conference of the IEEE Computer and Communications Societies, Proc. of IEEE*, vol. 2, pp. 775-784, March 2000.

[10] R. Stoleru, T. He, J. A. Stankovic "Range-free Localization," chapter in *Secure Localization and Time Synchronization for Wireless Sensor and Ad Hoc Networks*, editors: R. Poovendran, C. Wang, and S. Roy, *Advances in Information Security series*, vol. 30, Springer, 2007.

[11] T. He, C. Huang, B.M. Blum, J. A. Stankovic, and T. Abdelzaher, "Range-Free Localization Schemes for Large Scale Sensor Networks", *In Proc. of the ninth annual international conference on Mobile computing and networking (MobiCom 2003)*, San Diego, California, pp. 81-95, September 2003.

[12] C. Liu, T. Scott, K. Wu, and D. Hoffman, "Range-Free Sensor Localization with Ring Overlapping Based on Comparison of Received Signal Strength Indicator", *International Journal of Sensor Networks (IJSNet)*, vol. 2, no. 5, pp. 399-413, 2007.

[13] V. Vivekanandan, V. W. S. Wong, "Concentric Anchor-Beacons (CAB) Localization for Wireless Sensor Networks", *IEEE International Conference on Communications, ICC'06*, Istanbul, 2006, vol. 9, pp. 3972-3977.

[14] D. J. Dubois, H. Prade, *Fuzzy Sets and Systems: Theory and Applications*, Academic Press, 1980.